

Problem Set I - 1981 (January)

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The problems in this set are similar to the ones that students applying to universities and colleges usually solve at their entrance examinations. We recommend them to everyone who is applying. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Simplify the expression $x + x^3 + x^5 + \dots + x^{27}$.
2. The height of an isosceles trapezoid is 10 units. Find its area if its diagonals are perpendicular to each other.
3. A farm raises rabbits. At the beginning of three consecutive years, the farm buys 1000, 3000, and 3000 rabbits. At the end of the third year, there are 8261 rabbits on the farm. What is the average yearly percentage of growth?
4. Two vertices of an isosceles triangle are $(1, 6)$ and $(-5, 2)$. Where can the third vertex be?
5. Solve the given equation.

$$x^2\sqrt{\sin 2x} + 4 = 4\sqrt{\sin 2x} + x^2$$

6. Sketch the set of points in the coordinate plane whose coordinates satisfy the given equation.

$$\frac{y - x^4}{\left(\log \frac{9 - y}{5}\right) \sqrt{4x^2 - 4xy + y^2 - 9}} = 0$$

7. From one vertex of a triangle, we draw the perpendiculars to the interior and exterior angle bisectors of the other two vertices. Prove that the feet of these perpendiculars lie on a line.
8. Find the minimum value of the expression $y - 2x$ if

$$x^2 + 2y^2 = 8.$$