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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Is it true that the given number is rational?

$$\frac{\sqrt{3} \cdot \sqrt{6}}{3\sqrt{2} - \sqrt{32} + \sqrt{8}}$$

2. Find the equation of the image of the parabola $y = -2x^2 - 20x - 47$ under the reflection about the point $P(2, 1)$.
3. Let P be any point inside the base AB of the isosceles triangle ABC . Prove that the circles circumscribed around triangles ACP and BCP have equal radii.
4. Which real numbers x satisfy the given equation? (a is a real parameter.)

$$\frac{1 + a^2}{\sin x} = 4a \cos x$$

5. Which points of the plane have coordinates that satisfy the equation $x^4 - 2x^2y + y^2 - 1 = 0$?
6. Find all values of m for which the roots of the equation $x^4 - (3m + 2)x^2 + m^2 = 0$ form an arithmetic sequence.
7. Find the value of the expression

$$\sin(\sin \alpha) \cdot \cos(\sin \beta) + \cos(\sin \alpha) \cdot \sin(\sin \beta)$$

if

$$180^\circ < \frac{\alpha + \beta}{2} < 270^\circ, \quad 90^\circ < \frac{\alpha - \beta}{2} < 180^\circ, \quad \tan \frac{\alpha + \beta}{2} = 1, \quad \text{and} \quad \tan \frac{\alpha - \beta}{2} = -2\sqrt{2}$$

8. Find the point P in the plane of triangle ABC for which $PA^2 + PB^2 + PC^2$ is minimal.