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The problems in this set are similar to the ones that students applying to universities and colleges usually solve at their entrance examinations. We recommend them to everyone who is applying. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given system of equations.

$$\begin{aligned}x^3 - x^2y - 4xy^2 + 4y^3 &= 0 \\ 2x^2 + y^2 &= 12\end{aligned}$$

2. Two opposite vertices of rhombus $ABCD$ are $A(0, 0)$ and $C(2, 4)$. The angles at vertices A and C measure 120° . Find the coordinates of vertices B and D .

3. Solve each of the following equations.

a) $(4 - x^2)\sqrt{1 - x} = 0$

b) $(0.4^x - 2.5^{x+1} - 1.5)\log_3(x + 3) = 0$

c) $\sin x = 5 \cos \frac{x}{2}$

4. The perimeter of parallelogram $ABCD$ is 26 units, and $AB < AD$. The radius of the circle inscribed in triangle BCD is $\sqrt{3}$ units, and the angle at vertex B is 120° . Find the sides of the parallelogram.
5. Triangle ABC is an acute isosceles triangle with $AB = BC$. Its orthocenter is M , and $MB = AC = 1$ unit. Find the angles of the triangle and the length of the legs. (How many solutions does the problem have if we do not assume that the triangle is acute?)

6. Prove that in every triangle,

$$\sin^2 \frac{\alpha - \beta}{2} + \sin \alpha \sin \beta + \sin^2 \frac{\gamma}{2} = 1$$

where α , β , and γ are the angles of the triangle.

7. Solve the given inequality.

$$\log_{x-3}(x-1) < 2$$

8. Find all pairs of numbers (x, y) that satisfy each of the following equations.

a) $\sin^2(x + y) - \cos^2(x - y) = 1$

b) $\frac{1}{\sqrt{2}}(\sin x + \cos x) = 2y^2 - 4y + 3$