

Problem Set III -1982 (Part 2)

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The problems in this set are similar to the ones that students applying to universities and colleges usually solve at their entrance examinations. We recommend them to everyone who is applying. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation over the set of rational numbers.

$$\frac{2}{1-2x} + \frac{3}{2x+1} + \frac{6}{4x^2-1} = 1$$

2. Two medians of a triangle are 3 and 5 units long, and the angle between them is 65° . Find the area of the triangle.
3. The sum of three numbers is 71. If we increase the third number by 1, we get three consecutive terms of an arithmetic sequence. If instead we increase the first number by 3, we get three consecutive terms of a geometric sequence. Find the three numbers.
4. The center of a circle is on the line through the points $A(1, 1)$ and $B(7, 10)$. The circle is tangent to the line g with equation $x + 2y = 21$, at the point P whose x -coordinate is 5. Find the equation of the circle.
5. Solve the given equation over the real numbers.

$$\frac{2 \sin x}{2 \sin x - 1} + \frac{2 \sin x - 1}{2 \sin x} = \frac{5}{2}$$

6. The base of a right prism is a triangle with sides 4 cm, 5 cm, and 7 cm. The height of the prism is equal to the longest altitude of the base triangle. Find the volume of the prism.
7. Let M be a point inside the diagonal BD of rectangle $ABCD$. The line through M perpendicular to BD intersects line AB at E and line BC at F . Prove that
- a) $AB^2 \cdot ME = BC^2 \cdot MF$;
- b) the value of $\frac{AB \cdot ME + BC \cdot MF}{MB}$ does not depend on where M is on the segment BD .
8. Find all real numbers x , y , and z that satisfy the given system of equations.

$$\begin{aligned} x + y + z &= 2 \\ 2xy - z^2 &= 4 \end{aligned}$$