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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The base of an isosceles triangle is 24 units long, and its legs are 20 units long. How far is the centroid of the triangle from the center of its circumscribed circle?

2. Solve each of the following equations.

a) $\sqrt{\frac{x}{1+x}} + \sqrt{\frac{1+x}{x}} = \frac{5}{2}$

b) $\cot x + \frac{1}{\sin x} = \tan x$

c) $2 \log_4 (4-x) = 4 - \log_2 (-x-2)$

3. The hypotenuse of the right triangle ABC is $AB = 2$ units. Let B_1 be the midpoint of leg AC , and A_1 the midpoint of leg BC . Find the radius of the circle circumscribed around the quadrilateral ABA_1B_1 , if this quadrilateral is cyclic.

4. Consider the equation

$$(4x^2 - 4(a-2)x + a)(4x^2 - 4ax + 3a - 2) = 0$$

Find all values of the real parameter a for which every root of this equation is real.

5. Find the equation of the circle that is tangent to the line $2x + y = 19$ at the point with x -coordinate 7, and cuts chords of equal length from the two coordinate axes.

6. The perimeter of a right trapezoid is 4 units, and its acute angle is 45° . How should we choose the height of the trapezoid so that its area is as large as possible? What is the maximal area?

7. The sides of a triangle are a , b , and c . Prove that if

$$\frac{1}{a+b} + \frac{1}{b+c} = \frac{3}{a+b+c}$$

then the angle opposite side b is 60° . State the converse of this statement and prove it as well.

8. The circles k_1 and k_2 intersect at points A and B . The tangent line to k_1 at A intersects k_2 at C , and the tangent line to k_2 at B intersects k_1 at D . Prove that

$$AB^2 = AD \cdot BC \quad \text{and} \quad BD^2 \cdot BC = AC^2 \cdot AD$$