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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation.

$$\frac{\sqrt{x} + x}{\sqrt{x} - x} + \frac{\sqrt{x} - x}{\sqrt{x} + x} = \frac{4}{x} + \frac{4}{1 - x}$$

2. We connect a vertex of a rhombus with the midpoint of one of the opposite sides. This splits the rhombus into a triangle and an isosceles trapezoid. What is the ratio of the diagonals of the rhombus?

3. Solve the given equation.

$$\log_{2x}(6x^2 - x) = 3$$

4. In a right triangle, the altitude drawn from the right angle is on the line $y = 3x + 2$, and the median drawn from the right angle is on the line $y = 2x + 3$. The point $(6, 8)$ is on the line of the hypotenuse. Find the coordinates of the vertices.

5. Solve the given equation.

$$\tan x + \sqrt{3} \tan^2 x = \sqrt{3} \cot x + \cot^2 x$$

6. For which real numbers are the following functions defined? Also find the range of each function.

a) $\cos(\cos x)$

b) $\sqrt{\log_2(\sqrt{-2x} - \sqrt{-x})}$

7. The base of a pyramid is a square, and its apex is directly above the midpoint of one of the base edges. One of the lateral edges is twice as long as another one. Find the angles of the lateral faces.

8. The four zeros of the parabolas $y = x^2 + b_1x + c_1$ and $y = x^2 + b_2x + c_2$ form an arithmetic sequence. Prove that the two discriminants are either equal or one of them is nine times the other.