

author: Viktor Scharnitzky

translated from: KoMaL, March 1983, pages 105-106

Pdf file: KoMaL, March 1983, pages 105-106

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve each of the following equations.

a) $9\sqrt[3]{x} - 2 \cdot 3\sqrt[3]{x} = 3$

b) $\frac{100 - x^2}{10 - x} = |10 + x|$

c) $\frac{5}{3}\sqrt{15x} - \frac{3}{5}\sqrt{15x} - 11 = \frac{1}{3}\sqrt{15x}$

2. The longer one of the parallel sides of trapezoid $ABCD$ is $AB = 240$. The diagonal AC makes an angle of 30° with AB , it is perpendicular to side BC , and it bisects the angle DAB . Find the area of the trapezoid.

3. The endpoints of one of the diagonals of rectangle $ABCD$ are $A(-10, -6)$ and $C(9, 16)$, and the slope of the line AB is $\frac{5}{12}$. Find the coordinates of vertices B and D , and the area of the rectangle.

4. The sum of the first three terms of an arithmetic sequence is 15. If we square these terms, we get three consecutive terms of a geometric sequence. Find the common difference of the arithmetic sequence and the common ratio of the geometric sequence.

5. In triangle ABC , $AC = BC$. We choose points D and E on side AC so that $AD = DE = EC$. Find the area of the triangle if $BD = 8.5$ and $BE = 10$.

6. The angles α and β of a triangle satisfy

$$(1 + \tan \alpha)(1 + \tan \beta) = 2$$

Find the third angle of the triangle.

7. The circle k_2 with center A intersects the circle k_1 with diameter AB at points C and D . Let E be the point where k_2 intersects the segment AB . Let M be any point inside the arc CE of k_2 that is inside triangle ABC , and let N be the second intersection of the line BM with k_1 . Prove that $MN^2 = CN \cdot DN$.

8. From a four-digit number, we subtract the numbers we get by deleting its last digit, its last two digits, and its last three digits. The result is 1765. Find the number.