

author: Imre Rábai

translated from: KoMaL, December 1984, pages 439-440

Pdf file: KoMaL, December 1984, pages 439-440

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The bisector of the right angle of a right triangle divides the hypotenuse in a 1 : 3 ratio. In what ratio does the altitude belonging to the hypotenuse divide the hypotenuse?
2. Solve each of the following equations.

a) $2x - 3 = |3 - 2x|$ b) $\sqrt[3]{2^{2x-3}} = 8^{x-\frac{3}{2}}$ c) $\log_{2x-3} \sqrt{2x-3} = \frac{1}{2}$
3. The height of a frustum of a pyramid with a triangular base is 10 units, and the edges of its base are 27, 29, and 52 units long. The perimeter of the top face is 72 units. Find the volume of the frustum.
4. The endpoints of one of the diagonals of rhombus $ABCD$ are $A(-4, -5)$ and $C(5, 4)$, and the area of the rhombus is 27 square units. Find the coordinates of the other two vertices.
5. Solve the following systems of equations.

a) $\tan x + \tan y = 2\sqrt{3}, \quad \tan x \cdot \tan y = 3$
 b) $x + y = \frac{2\pi}{3}, \quad \tan x \cdot \tan y = 3$
6. The sides of a triangle are a, b, c , and the opposite angles are α, β, γ , respectively.

a) Find the angles of the triangle if $2(a + c) = 3b$ and $\gamma - \beta = 60^\circ$.
 b) Find the sides of this triangle if the radius of its circumscribed circle is 5 units.
7. Find the values of the parameter a for which the system of equations

$$\begin{aligned} x + y &= -2 - xy \\ x - y &= a + axy \end{aligned}$$

has exactly one solution.

8. Find the largest possible value of ab if

$$0 < a < 1, \quad 0 < b < 1, \quad \text{and} \quad \log_{\frac{1}{2}} a \cdot \log_{\frac{1}{2}} b = 1$$