

author: János Rácz

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Consider the circles $K_1: (x - 5)^2 + (y + 2)^2 - 17 = 0$ and $K_2: (x + 1)^2 + (y - 5)^2 - 68 = 0$. Let C be the center of K_1 , D the center of K_2 , and let A and B be the intersection points of the two circles. Find the area of quadrilateral $CADB$.

2. For which values of the real parameter k does the quadratic equation

$$(k + 2)x^2 - 2(k - 11)x + (k - 3)(k + 5) = 0$$

have two real roots with opposite signs?

3. Solve the given equation.

$$\sqrt{x + 20} - \sqrt{x - 15} = x - 9$$

4. Consider the circle $K: (x + 3)^2 + (y - 2)^2 - 19 = 0$ and the line $L: 11x - 2y + 16 = 0$. Find the point P for which the line connecting the points of tangency of the two tangent lines drawn from P to the circle K is the given line L .

5. Solve the given equation.

$$2 \cos \frac{x}{2} + \sqrt{\cos 2x - 3 \cos x} = 0$$

6. Consider a triangle ABC . Let D be the third vertex of the equilateral triangle drawn outward on side AC , and let E be the vertex with the 120° angle of the isosceles triangle with a 120° angle drawn outward on side BC . Let F be the midpoint of side AB , and let G be the reflection of E about F . What kind of triangle is DEG ?

7. Solve the given inequality.

$$2 \cos x \sin \frac{x}{2} + \sqrt{1 - \cos x} \geq 0$$

8. Solve the given inequality,

$$\log_a b + \log_b a \leq 2, \quad \text{where} \quad a = \frac{x^2 - x + 20}{50} \quad \text{and} \quad b = \frac{x^2 - 8x + 17}{50}$$