

Problem Set II - 1984 (January 1984)

author: Imre Rábai

translated from: KoMaL, January 1984, pages 6-7

Pdf file: KoMaL, January 1984, pages 6-7

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Three numbers are consecutive terms of an arithmetic sequence, and the middle one is 2. Find the other two numbers if the squares of the three numbers are consecutive terms of a geometric sequence.
2. a) Find the exact value of $\tan 15^\circ$.
b) The legs a and b of a right triangle satisfy $(a - b)^2 = 2ab$. Find the exact values of the acute angles of the triangle.
3. Consider the equation $x^2 + mx - 2m^2 + 3m - 1 = 0$, where m is a real parameter.
a) Find the values of m for which one root of the equation is twice the other root.
b) Find the value of m for which the product of the two roots is as large as possible.
4. Find the equation of the circle that is tangent to the line $x + y = 0$ at the origin, and is also tangent to the line $x = 1$.
5. The circles k_1 and k_2 are tangent to each other from outside at point A . A common tangent line of the two circles touches k_1 at E_1 and k_2 at E_2 . Find the radii of the circles if $AE_1 = 8$ and $AE_2 = 6$ units.
6. For which real numbers x is the value of the expression

$$\frac{3x^2 - 7x + 8}{x^2 + 1}$$

greater than 1 and at most 2?

7. The radius of the base of a right circular cone is equal to its height. We inscribe a right circular cylinder in the cone so that the height of the cylinder is equal to the diameter of its base. What percent of the surface area of the cone is the surface area of the cylinder? What percent of the volume of the cone is the volume of the cylinder?
8. Find the values of the parameter m for which the equation

$$\tan 2x = m \cot x$$

has a solution. Solve the equation if $m = 1$.