Problem Set I - 1985

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, student assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve each of the following equations.

   a) $2^{2x^2+6x-8} = 4^{\frac{x-1}{x+4}}$ b) $\sqrt{x^2+3x-4} = \left(\dfrac{x-1}{x+4}\right)^{\frac{1}{2}}$ c) $\log\left(x^2+3x-4\right) = \log\left(\dfrac{x-1}{x+4}\right)$

2. Compute the exact value of each of the following expressions.

   a) $\tan 110° + \cot 20°$ b) $\dfrac{\sqrt[3]{9^2}\left(\frac{1}{3}\right)^6 \cdot 3\left(\sqrt[3]{3}\right)^4}{\left(\sqrt[3]{3}\right)^{-1} \cdot 27^{-\frac{2}{3}}}$ c) $27^{\frac{1}{\log_5 3}} - 10^{2-\log 4}$

3. The base of a straight prism is triangle ABC, in which $AC = 3$ unit and $BC = 5$ unit, and the angle at vertex C is $120°$. The face with the greatest area is 35 unit^2. Compute the volume and surface area of the prism.

4. The vertices of triangle ABC are given as $A(2,0)$, $B(-2,0)$, and $C(1,3)$. The circle $x^2+y^2=4$ intersects side AC in point R, and intersects the side BC in point S. Find the equation for the circle that contains points C R and S.

5. Find the quadratic equation in which the quadratic coefficient is 1, one of the solution is 2, and the other solution equals to the discriminant of the equation.

6. Prove that if $\sin(\alpha+\beta) = 0$, then $\cos(\alpha+2\beta) = \cos\alpha$. Is the converse of the statement true?

7. The height belonging to the base in an isosceles triangle is 8 units long. The circle's inscribed circle has a radius of 2 units long. Find the radius of the circle that is tangent to the legs and the inscribed circle.

8. Solve the equation
$$(a-4)\cdot 2^x + a \cdot 2^{-x} = 2a$$
   where a is a real parameter.

