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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve each of the following equations over the real numbers.

a) $(3x^2 + 2x - 8) \cdot \sqrt{x^2 + 2x - 3} = 0$

b) $\sqrt{3x^2 + 2x - 8} \cdot \sqrt{x^2 + 2x - 3} = 0$

2. The diagonals of a parallelogram are 8 and 12 units long, and its area is 40 square units. Find the lengths of the sides of the parallelogram.

3. The sum of the first four terms of the geometric sequence (a_n) is 16, and $2(a_4 - a_1) = 7(a_3 - a_2)$. Find the first term and the common ratio of the sequence.

4. Solve the given system of trigonometric equations over the real numbers.

$$\begin{aligned}\sin(x - y) &= 2 \sin x \sin y \\ x + y &= \frac{\pi}{2}\end{aligned}$$

5. Find the equation of the chord of the parabola $y = x^2$ whose midpoint is $P(1, 6)$.

6. Among the rectangles with perimeter 60 units, find the area of the one whose diagonals are as short as possible.

7. Solve the given inequality, where a is a real parameter.

$$\log_a(x - a) > \log_{\frac{1}{a}}(x + a)$$

8. Let a , b , and c be the sides of a triangle, and α , β , and γ the opposite angles, respectively. Prove that

$$\begin{aligned}a^2 + b^2 - ab\sqrt{2} \cos(\gamma + 45^\circ) &= b^2 + c^2 - bc\sqrt{2} \cos(\alpha + 45^\circ) \\ &= c^2 + a^2 - ca\sqrt{2} \cos(\beta + 45^\circ)\end{aligned}$$