Economics Entrance Exam - 1986

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These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1986. Problems 6/a, 7/a, and 8/a were recommended for students coming from vocational secondary schools (in place of problems 6, 7, and 8).

1. Find n if the ratio of the sum of the first n positive even numbers and the sum of the first n positive odd numbers is $\frac{21}{20}$.

2. Solve the given system of equations over the real numbers.

$$\begin{aligned} 8^{2x+1} &= 32 \cdot 2^{4y-1} \\ 5 \cdot 5^{x-y} &= \sqrt{25^{2y+1}} \end{aligned}$$

3. For which real numbers are the following expressions defined?

   a) $\log\log\sqrt{x}$ b) $\log\sqrt{\log x}$ c) $\sqrt{\log\log x}$

4. In rectangle $ABCD$, $AB = 2.4 \cdot BC$. We enlarge the rectangle by a dilation with center A so that the area of the new rectangle is $\frac{9}{4}$ times the area of the original one. Because of the enlargement, the diagonal of the rectangle becomes 13 units longer. Find the sides of rectangle $ABCD$.

5. The base ABC of the triangular pyramid $OABC$ is an isosceles right triangle with $\angle ACB = 90^\circ$. The lateral edge $OC = 3\sqrt{3}$ is perpendicular to the plane of the base. The face OAB makes an angle of 60° with the plane of the base. Find the surface area and the volume of the pyramid.

6. Let $ABCD$ be a quadrilateral in the plane. Give a construction for the point P for which $\overrightarrow{PA} + \overrightarrow{PB} + \overrightarrow{PC} + \overrightarrow{PD} = \mathbf{0}$.

7. How should we choose p and q in the polynomial $x^2 + px + q$ if we want to get the polynomial $x^4 + 1$ when we multiply it by a suitably chosen quadratic polynomial?

8. Solve the given equation over the set of integers.

$$\cos\left(\frac{\pi}{4}\left(3x - \sqrt{9x^2 - 16x - 80}\right)\right) = 1$$

6/a. A parabola with an axis parallel to the y-axis, opening upward, passes through the point $A\,(5, 4)$ and is tangent to the x-axis. The tangent line of the parabola at A is perpendicular to the vector $\mathbf{v}\,(4, -1)$. Find the equation of the parabola.

7/a. Prove that a triangle is a right triangle if and only if its acute angles α and β satisfy $\sin\alpha + \sin\beta = \cos\alpha + \cos\beta$.

8/a. Prove that every term of the sequence

$$a_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{2n}$$

is greater than the previous term, but less than 1.

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