Entrance Exam for Universities Abroad - 1986

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These problems were given in 1986 at the written mathematics entrance examination of Hungarian students applying to study at universities abroad. Problems 1–5 were the same for everyone. Of problems 6–8, the ones marked A were recommended for students who graduated from academic high schools, and the ones marked B for students who graduated from vocational secondary schools.

1. Solve the given equation over the real numbers.

$$\log\left(36^{5x}+792\right)=\log 16+\log 63$$

2. In a right triangle, the bisector of one of the acute angles divides the opposite side into two segments of length 1 and 2. Find the angles and the sides of the triangle.

3. Solve the given system of equations over the real numbers.

$$\sqrt{x+3y+1}=2, \qquad \sqrt{2x-y+2}=7y-6$$

4. Three vertices of parallelogram $ABCD$ are $A\,(-3,-2)$, $B\,(2,0)$, and $C\,(3,3)$. The line through B perpendicular to the diagonal AC intersects the line CD at E. Find the coordinates of E.

5. Of four numbers, the first three are consecutive terms of a geometric sequence, and the last three are consecutive terms of an arithmetic sequence. The sum of the first and the fourth number is 14, and the sum of the second and the third is 12. Find the four numbers.

6A. Solve the given system of inequalities over the real numbers.

$$x^2-6x-7<0, \qquad \sin x>0$$

7A. Let $ABCD$ and $EFGH$ be two parallelograms in the same plane. Prove that the midpoints of the segments AE, BF, CG, and DH, if they are not on one line, are the vertices of a parallelogram.

8A. Prove the following statement for every positive integer n: if a, b, c are integers such that n divides $a+b+c$ and $a^2+b^2+c^2$, then n also divides $a^4+b^4+c^4$.

6B. In the isosceles triangle ABC, the base BC is 10 units long, the altitude from A is 12, and D is the midpoint of the base. Let M be a point of the leg AC. Find the length of the segment AM if the quadrilateral $AMDB$ is cyclic.

7B. Solve the given system of equations over the real numbers.

$$\frac{xy}{x+y}=1-z, \qquad \frac{yz}{y+z}=2-x, \qquad \frac{zx}{z+x}=2-y$$

8B. Prove that if $0\le x<\dfrac{\pi}{2}$, then

$$\frac{\sin x}{1+\tan x}\le\frac{\sqrt{2}}{4}$$

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