Technical Colleges Entrance Exam - 1986

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These problems were given at the written mathematics entrance examination of the technical colleges in Hungary in 1986. Problems 6/a, 7/a, and 8/a were recommended for students coming from vocational secondary schools (in place of problems 6, 7, and 8).

1. Solve the given equation over the real numbers.

$$\log(4x+2) - \log(-6x) = \log(1-2x)$$

2. The first term of an arithmetic sequence is -210, and its nth term is 228. The sum of the terms between them is 45. Write down the first n terms of the sequence.

3. In triangle ABC, the angle at vertex A is a right angle, and one of the angles is 30°. The distance of vertex A from the center of the inscribed circle is $2\sqrt{2}$. Find the radius of the circumscribed circle.

4. For which real numbers x are the following expressions defined? What values do they take?

   a) $\sin\left(\frac{\pi}{6}\cos(\pi x)\right)$ b) $\sqrt{\sin(\pi x)}$

5. Two sides of a triangle are 8 and 12, and the median drawn to the third side is 9. Find the area of the triangle.

6. Solve the given inequality over the real numbers.

$$\left(x^2 - x + 1\right)^{x-2} > 1$$

7. The base of a regular square pyramid is the square $ABCD$, its apex is O, and all of its edges are h units long. Find the area of the quadrilateral that the plane through the base edge AB and the midpoint F of the edge OC cuts out of the pyramid. What angle does this plane make with the base?

8. We write the consecutive positive integers on a board, starting with 1, up to a certain number. Then we erase one of the numbers. The arithmetic mean of the remaining numbers is $\frac{602}{17}$. Which number did we erase?

6/a. The height of a trapezoid inscribed in a circle is 8, and one of its legs is seen from the center of the circle at an angle of 60°. Find the area of the trapezoid.

7/a. The line g through the origin cuts a chord of length 10 from the circle $(x-5)^2 + (y-10)^2 = 50$. Find the equation of g and the coordinates of the endpoints of the chord.

8/a. Solve the given equation over the real numbers.

$$\sqrt{x+2+4\sqrt{x-2}} + \sqrt{x+7-6\sqrt{x-2}} = 5$$

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