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1. Two sides of an obtuse triangle are 4 and 5 units long. The sine of the angle between these sides is 0.38. Find the measures of the other two angles.
2. Solve each of the given equations over the real numbers.

a) $x^2 + \frac{1}{x-2} = 2x - \frac{1}{2-x}$

b) $\ln(1 + \sin x) = 0.31$

3. In an arithmetic sequence $a_p = q$ and $a_q = p$. Express a_n in terms of p , q , and n .
4. One leg of a right triangle is the geometric mean of the hypotenuse and the other leg. Find the ratio of the radii of the inscribed and circumscribed circles.
5. Solve the given equation over the real numbers.

$$\sin^6 x + \cos^6 x = \frac{7}{16}$$

6. Consider a circle with radius R . Let P be a point on the circle. We draw chords PA , PB , PC so that $\angle APB = \angle BPC = 60^\circ$. Find the area of triangle ABC .
7. In triangle ABC , let E_1 denote the point where the inscribed circle is tangent to side BC . Let E_2 denote the point where side BC is tangent to the circle that is tangent to side BC from outside and to the extensions of the other two sides. (This second circle is called the excircle drawn to side BC .) Express the distance E_1E_2 in terms of the sides of the triangle.
8. Consider the parabola with equation $y = ax^2 + bx + c$. This parabola passes through the points $(0, k)$, $(k, 2k)$, and $(2k, 0)$. Find all values of k for which the parabola intersects the x -axis in $(-1, 0)$. In this case, what are the values of a , b , and c ?