

Problem Set I - 1987

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given inequality over the real numbers.

$$(x - 1)^2 + 2 \leq 2\sqrt{x^2}$$

2. Find the equation of the circle that passes through the point $A(2, 1)$, is tangent to the x -axis, and whose center is on the line $x - 2y = 1$.

3. Find the exact value of $\sin x$ if $\tan 2x + 2 \tan x = 0$.

4. Solve the given equation over the real numbers.

$$x^{2 - \log \frac{x}{2}} = 4$$

5. In triangle ABC , $BC = \sqrt{10}$ units, the radius of the circumscribed circle is $r = \sqrt{5}$ units, and $AB = 2\sqrt{2} \cdot AC$. Find the length of the side AB .

6. In rectangle $ABCD$, $AB = 2 \cdot BC$. The distances of the vertices C and D from a point P of the line AB are 5 and $\sqrt{41}$ units. Find the lengths of the sides of the rectangle.

7. Solve the given system of equations, where a is a real parameter.

$$\begin{aligned}x^2 + y^2 &= 16 \\(x - 2a)(ay + x) &= 0\end{aligned}$$

8. The real numbers x , y , and z satisfy the following equations.

$$\begin{aligned}x + y &= z - 1 \\xy &= z^2 - 7z + 14\end{aligned}$$

For what value of z is the sum $x^2 + y^2$ the largest? What is the largest value of $x^2 + y^2$?