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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation.

$$x^2 - 5x + 6 = 1 - \frac{1}{x^2 - 5x + 7}$$

2. In rhombus $ABCD$, $AC = AB$. We draw the perpendiculars from vertex A to the sides DC and BC , with feet M and N . Let F and G be the points where the lines AM and AN intersect the diagonal BD . Prove that $FG : MN = 2 : 3$.

3. Without using approximate values, decide which number is greater:

$$A = \left(\sqrt{2} - \log_2 \sin \frac{29\pi}{6} \right)^2 \quad \text{or} \quad B = \frac{\sqrt{8}}{2 - \sqrt{2}}$$

4. Two circles with different radii are tangent to each other from outside at point A . The line through their centers intersects the smaller circle again at B , the larger circle again at C , and one of their common external tangent lines at P . Find the radii of the circles if $PB = 8$ and $PC = 18$.

5. For which real numbers x is it true that

$$\log_3 (9^{2x} - 3^{2x+1} + 3) < 2 \log_9 7 ?$$

6. Prove that for any point inside a regular tetrahedron with edges $\frac{\sqrt{6}}{2}$ units long, the sum of the reciprocals of its distances from the faces is at least 16.

7. Let S_n be the sum of the first n terms of a geometric sequence, and Q_n the sum of the reciprocals of the first n terms. Find the product of the first n terms.

8. How many four-digit numbers between 1000 and 9999 have all different digits and the absolute value of the difference of the first and last digits equal to 2?