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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation.

$$\log_{x-4} (x-4)^7 - 2x - \frac{1-3x}{7} = 2 - \frac{2x-1}{3}$$

2. The parallel sides of a trapezoid are 4 and 7 units long, and its legs are 4 and 5 units long. How far is the intersection of the diagonals from the sides of the trapezoid?

3. Graph each of the following.

a) the function f for which $1 = \frac{2x+1}{f(x)}$ b) $x \mapsto 10^{\frac{1}{2} \log \sin^2 x}$ c) $x \mapsto \log_{1986} \sqrt{\frac{1+\cos x}{2 \cos^2 \frac{x}{2}}}$

4. In what base is the number 1331 a perfect cube?

5. Prove that $x + y + xy \leq x^2 + y^2 + 1$ for all real numbers x and y .

6. The sides of rectangle $ABCD$ are $AB = CD = 38$ and $BC = AD = 51$ units. The points E and F are on side AD so that $AE = FD = 3$ units. From which point of the broken line $AB-BC-CD$ is the segment EF seen at the largest angle?

7. The points A, B, C, D, E form a pyramid whose base is the rhombus $BCDE$ and whose apex is A , and the edge AE is perpendicular to the plane $BCDE$. We know that $AE = 6$, $BC = 8$, and $\angle BCD = 120^\circ$.

- a) What angle does the edge AC make with the plane of the base?
- b) What is the angle between the planes ACB and ACD ?
- c) How far is the edge AC from the segment BD ?

8. Let $f(x) = x^2 + x + 2$ and $g(x) = x^2 - x + 2$. Prove that there is no function h , defined for every x , for which

$$h(f(x)) + h(g(x)) = g(f(x))$$

for every x .