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translated from: KoMaL, March 1987, page 105

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given system of equations.

$$xy(x^2 + y^2) = -742, \quad x^4 + y^4 = 2417$$

2. The equations of two side lines of a triangle are $5x - 13y - 26 = 0$ and $4x + 3y + 27 = 0$. Find the equation of the line that passes through the point $P(1, 3)$ and forms a triangle of the smallest possible area with the two given lines.
3. The faces of a tetrahedron are congruent triangles, and its edges are 10, $3\sqrt{13}$, and $\sqrt{145}$ units long. Find the volume of the tetrahedron.

4. Solve the given equation.

$$\frac{\log_x(x^2 + x + 3)}{4x^2 + 7} + \frac{4x^2 + 7}{\log_x(x^2 + x + 3)} = \frac{2x^2 + 1}{x^2 + 1}$$

5. In a chess tournament with masters and grandmasters, everyone plays everyone else twice, once with white and once with black. The total score of the masters is 1.2 times the total score of the grandmasters. How many grandmasters and masters took part in the tournament if there were three times as many masters as grandmasters?
6. Solve the given inequality.

$$x + 21 - 30\sqrt{\frac{x + 21}{x - 3}} - \frac{175}{x - 3} \geq 0$$

For which values of x does equality hold?

7. Solve the given equation.

$$\sqrt{\frac{(1 - \sqrt{3})(2 \cos x + 1)}{8}} + \cos \frac{x}{2} = 0$$

8. Solve the given inequality.

$$\sqrt{\frac{(1 - \sqrt{3})(2 \cos x + 1)}{8}} + \cos \frac{x}{2} \geq 0$$