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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation over the real numbers.

$$3 \log_{1000} (x - 1) + 2 \log_{100} (x + 3) - 2^{x+3} \cdot \log_{10} (x^2 + 2x - 3) = 0$$

2. Let the function $f : \mathbb{R} \rightarrow \mathbb{R}$ be defined as follows: for every real number x , $f(x)$ is the distance of x from the integer in the closed interval $[0, 2]$ that is farthest from x . Find a formula for f and graph it.
3. The hypotenuse of a right triangle is c , and its legs are a and b . We rotate the triangle around each of its sides. Let V_c be the volume of the solid we get by rotating around the hypotenuse, and V_a, V_b the volumes of the solids we get by rotating around the legs. Prove that

$$\frac{1}{V_c^2} = \frac{1}{V_a^2} + \frac{1}{V_b^2}$$

4. Which real numbers x satisfy the given equation?

$$\sin^3 x (1 + \cot x) + \cos^3 x (1 + \tan x) = \cos 2x$$

5. The side of square $ABCD$ is a . We choose a point E on the diagonal AC so that $AE : AC = 1 : 3$. The line DE intersects the circle circumscribed around the square at F , and the extension of side CB at G . Prove that

a) GA is tangent to the circle;

b) $GF \cdot GD = 2a^2$.

6. In the tetrahedron $ABCS$, the edges SA, SB, SC are pairwise perpendicular. Is triangle ABC acute?
7. Starting with some integer, we take every eighth integer until the sum of the numbers taken is 473. Which number did we take first, and how many numbers did we take?
8. Find the integer values of the parameter k for which the roots of the equation

$$(k^2 - 1)x^2 - (2k^2 - 3k - 1)x + k^2 - 3k + 2 = 0$$

are integers.