Economics Entrance Exam - 1987

translated from: KoMaL, November 1987, pages 361-362
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These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1987. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all applicants

1. Find the exact value of the given expression without using a calculator or tables.

$$\frac{5 \cdot 8^{-\frac{2}{3}} + \frac{2}{\sqrt{3}} \left(\sin \frac{16\pi}{3} \right)^3}{\log_4 2}$$

(9 points)

2. The vertices A and B of parallelogram $ABCD$ are $A\,(1,4)$ and $B\,(6,6)$. The point $P\,(10,18)$ is on the line BC, and the point $R\,(-1,11)$ is on the line CD. Find the perimeter of the parallelogram. (11 points)

3. The base of an isosceles triangle is 36 units long, and its legs are 30 units long. How far is the centroid of the triangle from the center of its circumscribed circle? (12 points)

4. Find the largest subset of the real numbers on which the following expressions are defined.

   a) $\dfrac{\sqrt{x^2+4x-5}}{|x+2|-3}$ (8 points)

   b) $\log\left(1-\sin 2x\right)$ (5 points)

5. The base of a triangular pyramid is an equilateral triangle, and its lateral faces are congruent isosceles triangles. The cross-section through one lateral edge, perpendicular to the opposite base edge, has an area of $150\,\mathrm{cm}^2$. The volume of the pyramid is $1500\,\mathrm{cm}^3$. What angle does each lateral face make with the base? (13 points)

Recommended for students from academic high schools

6. Sketch the set of points $P\,(x,y)$ in the coordinate plane whose coordinates satisfy the following conditions.

   a) $|x| \le |y|$ (5 points)

   b) $|x+1|+|y+1| \le 2$ (8 points)

7. Solve the given system of equations over the real numbers.

$$\begin{aligned} x^{\log_8 y} + y^{\log_8 x} &= 4 \\ \log_4 x - \log_4 y &= 1 \end{aligned}$$

(14 points)

8. Let O be a point in the plane, and let $\overrightarrow{OA}$, $\overrightarrow{OB}$, $\overrightarrow{OC}$ be unit vectors such that O is outside the triangle ABC. Prove that

$$\left| \overrightarrow{OA} + \overrightarrow{OB} + \overrightarrow{OC} \right| > 1$$

(15 points)

Recommended for students from vocational secondary schools

6. Solve the given inequality over the real numbers.

$$2\log_x \frac{1}{4} + \log_4 \frac{1}{x} \leq -3$$

(13 points)

7. In triangle ABC, the median AA_1 is perpendicular to AB. Let a, b, c be the lengths of the sides BC, CA, AB. Express the cosine of the angle CAB in terms of b and c, and the cosine of the angle BCA in terms of a and b. (14 points)

8. Let a and b be real numbers with $a > b$ and $ab = 1$. Prove that

$$\frac{a^2 + b^2}{a - b} \geq 2\sqrt{2}$$

(15 points)

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