Entrance Exam for Universities Abroad - 1987

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These problems were given in 1987 at the written mathematics entrance examination of Hungarian students applying to study at universities abroad. Problems 1–5 were the same for everyone. Of problems 6–8, the ones marked A were recommended for students who graduated from academic high schools, and the ones marked B for students who graduated from vocational secondary schools.

1. The numerators of two fractions are both 3, and the sum of their denominators is 13. The sum of the two fractions is $\frac{39}{40}$. Find the two fractions.
2. Solve the given system of equations over the real numbers.

$$7^{x+1} \cdot 2^y = 4, \qquad \frac{1}{x-1} - \frac{1}{y} = \frac{4}{y\,(x-1)}$$

3. The diagonals of an isosceles trapezoid are perpendicular, and they divide each other into two parts in a $1:7$ ratio. Find the area of the trapezoid if its perimeter is $180\sqrt{2}$.
4. Two circles with radii R and r $(R > r)$ are tangent to each other from outside. Express, as a function of the radii, the length of the segment of their common internal tangent line that lies between the two common external tangent lines.
5. The sum of the first, third, and fifth terms of a geometric sequence is 182, and the sum of the second and fourth terms is 20 times the common ratio. Write down the first five terms of the sequence.

6A. Let ABC and DEF be any two triangles, and let S and T be their centroids. Prove that $\overrightarrow{AD} + \overrightarrow{BE} + \overrightarrow{CF} = 3\overrightarrow{ST}$.

7A. Solve the given inequality over the real numbers.

$$\log_{|x-1|} x \geq 1$$

8A. Is there a line in the plane all of whose points $P\,(x, y)$ satisfy the inequality $x^4 - 2x^2y + y^2 - 1 \leq 0$?

6B. In trapezoid $ABCD$, the base AB is twice as long as the parallel side CD and twice as long as the leg AD. The other leg BC is 5 units long, and the diagonal AC is 12 units long. Find the area of the trapezoid.

7B. Find the solutions of the following inequalities in the interval $\left[\frac{\pi}{4}, \pi\right]$.

a) $\dfrac{1}{\sin^2 x} \leq \dfrac{1}{\sin 2x - \cos^2 x}$ b) $\log_2 \sin 2x < -1$

8B. Let P be a point on the circle circumscribed around the equilateral triangle ABC. Prove that the sum $PA^2 + PB^2 + PC^2$ is constant, and find its value.

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