Technical Universities Entrance Exam - 1987

translated from: KoMaL, October 1987, pages 291-292
Pdf file: KoMaL, October 1987, pages 291-292

These problems were given at the written mathematics entrance examination of the technical universities and colleges in Hungary in 1987. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all applicants

1. If the sum of 101 consecutive odd numbers is $12\,827$, what are the smallest and the largest of them? (9 points)

2. Solve the given system of equations over the real numbers.

$$x^2 - y^2 = \frac{5}{2}, \qquad xy = \frac{\sqrt{6}}{2}$$

(10 points)

3. The reflection of the origin O about the point $A\,(5,5)$ is O_1, the reflection of O_1 about the point B is O_2, and the reflection of O_2 about the point $C\,(1,7)$ is O again. Find the coordinates of B, and prove that the quadrilateral $OABC$ is a rhombus. (13 points)

4. In the acute triangle ABC, side AB is 16 units long, and the altitude belonging to it is 12. We inscribe an isosceles right triangle in triangle ABC so that its hypotenuse is parallel to AB, its right-angle vertex is on AB, and its other two vertices are on AC and BC. Find the sides of the inscribed triangle. (13 points)

5. Solve the given equation over the real numbers.

$$\sin x + \cos x = \sin 2x - \frac{1}{2}$$

(13 points)

Recommended for students from academic high schools

6. For which x do the following expressions take positive values?

   a) $2^x + 2^{-x} - \dfrac{17}{4}$ b) $2 + \log_{1/3}(5x - 1)$ (13 points)

7. In rectangle $ABCD$, side AB is three times side BC. The distances of an inside point P from the vertices B, A, D are $PB = 4\sqrt{2}$, $PA = \sqrt{2}$, and $PD = 2$. Find the area of the rectangle. (14 points)

8. Prove that in every triangle, the midpoint of the segment connecting the center of an excircle (a circle tangent to the side lines from outside) and the center of the inscribed circle lies on the circumscribed circle. (15 points)

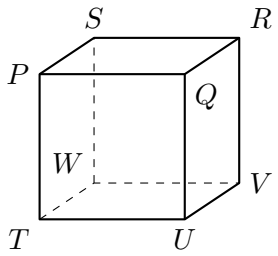
Recommended for students from vocational secondary schools

6. Prove that if the difference of the squares of the roots of the quadratic equation $ax^2 + bx + c = 0$ is $\dfrac{c^2}{a^2}$, then

$$b^4 - c^4 = 4ab^2c$$

(13 points)

7. The vertices of a cube are labeled as in the figure. Let A be the center of face $TUVW$, and B the center of face $UVRQ$. The edge UV intersects the plane PAB at X. Find the value of the ratio $\dfrac{UX}{XV}$.



(14 points)

8. The real numbers x, y, z satisfy

$$x + y = z - 1, \qquad xy = z^2 - 7z + 10$$

For which values of x, y, z is the sum $x^2 + y^2$ maximal? (15 points)

Downloaded from martahidegkuti.com