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Pdf file: KoMaL, November 1988, page 363

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equation and inequalities.

a) $\sqrt{4x + x^2} = 4 - x$ b) $\sqrt{4x + x^2} < 4 - x$ c) $\sqrt{4x + x^2} > 4 - x$

2. The parallel sides of a trapezoid are 1 and 4 units long, and its legs are 2 and 3 units long. Find the lengths of the diagonals of the trapezoid.

3. Solve the given system of equations.

$$\begin{aligned}x^2 - xy - 2y^2 &= 0 \\2x^2 + y^2 + \sqrt{2x^2 + y^2} &= 12\end{aligned}$$

4. For what values of the parameter a are the roots of the equation

$$x^2 - 2ax + 2a^2 - 3a = 0$$

real, and for which of these a does $(x_1 - x_2)^2$ take its largest value, where x_1 and x_2 are the roots of the given equation?

5. The vertex C of the square $ABCD$ is on the y -axis, and the opposite vertex is $A(2, 2)$. The area of the square is 20 square units. Find the coordinates of the unknown vertices of the square.

6. Solve the given equation.

$$3(\cot x - \tan x) = 2 \tan 2x$$

7. The first term of a geometric sequence is 3, the sum of the first n terms is 33, and the sum of the reciprocals of the first n terms is $\frac{11}{48}$. Find the first n terms of the sequence.

8. A cube is inscribed in a regular square pyramid so that four vertices of the cube are on the base of the pyramid, and the other four vertices are on the lateral edges of the pyramid. Prove that

$$V_1 \leq \frac{4}{9}V$$

where V_1 is the volume of the cube, and V is the volume of the pyramid. When does equality hold?