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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation over the real numbers.

$$3^x \cdot 4^{\frac{x}{x+1}} = 6$$

2. Find the equation of the circle that passes through the points  $A(0, 1)$  and  $B(0, 3)$  and is tangent to the  $x$ -axis.
3. The product of the first and fifth terms of a geometric sequence is 144, and the difference of the fourth and second terms is 18. Find the first term and the ratio of the sequence.
4. Consider the right circular cones for which  $b + 2r = 8$ , where  $b$  is the slant height of the cone, and  $r$  is the radius of its base. Which of these cones has the largest lateral surface area? Find the volume of this cone.
5. The numbers  $p$  and  $q$  are real numbers such that the roots of the equations  $4x^2 - 4px + q = 0$  and  $4x^2 - 4p^2x - pq = 0$  are real, and the roots of the second equation are 1 greater than the roots of the first equation. Find the values of  $p$  and  $q$ .
6. A convex hexagon can be inscribed in a circle of radius  $r$  so that three of its sides are 2 units long and the other three sides are 5 units long. Find the radius of the circle.
7. For which real numbers  $p$  does the equation  $\log_{1-x}(px) = 2$  have two different real solutions?
8. Find all pairs of numbers  $(x, y)$  that satisfy the given equation.

$$4 \sin^2 x - 4 \cos x \sin y - 5 = 0$$