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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The first digit of a four-digit number is 4. If we delete its last digit and write this digit in front of the remaining three digits, we get a number that is 2826 greater. What is this four-digit number?
2. In a triangle, $\tan \frac{\alpha}{2} = \frac{5}{6}$ and $\tan \frac{\beta}{2} = \frac{20}{37}$. Find $\tan \frac{\gamma}{2}$. Show that $b = \frac{a+c}{2}$.
3. A ship leaves P for Q , going upstream on a river, and at the same time a boat leaves Q for P . They meet when the boat has covered one third of its way. The ship waits 1 hour in Q , then turns back, and arrives in P 1 hour before the boat. If the boat doubled its speed (in still water), then it would arrive in P 3 hours before the ship. How long did it take for the ship to get from P back to P ?
4. Solve the given equation over the real numbers.

$$(x^2 - 1988^2)^2 - 7952x - 1 = 0$$

5. The elements of a set C are natural numbers. The least common multiple of the elements is 2210. No two elements of the set are relatively prime. The product of the elements is not divisible by 416. At most how many elements can C have, and what are these numbers?
6. For what values of the parameter k does the quadratic equation

$$(k+10)x^2 - 2(k+10)(k-5)x + (k-5)(k-18) = 0$$

have two different roots with the same sign?

7. Out of 66 actors, any two have already played together either in a theater performance, or in a film, or in a TV play, or in dubbing, but each pair played together in only one kind of production. Prove that there are three actors among them such that any two of them played together in the same kind of production.
8. Prove that in every triangle,

$$r_a + r_b + r_c - r_0 = 4R$$

where r_a, r_b, r_c are the radii of the excircles tangent to the sides a, b, c , r_0 is the radius of the inscribed circle, and R is the radius of the circumscribed circle.