Problem Set II - 1988 (December 1988)

author: Imre Rábai
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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The sides of triangle ABC are $AB = 13$, $BC = 17$, and $AC = 24$ units long. Let D be the point on the side AC for which $AD = 16$ units. Find the length of the segment BD.

2. Solve the given equation.
$$\left|x^2 - 2x - 1\right| = \frac{5x + 1}{3}$$

3. Find the exact value of $\sin 3x$ if $\cos 2x = -\frac{1}{4}$.

4. Solve the given inequality.
$$\log_{\frac{1}{2}}\left(5^{x+1} - 25^x\right) \geq -2$$

5. The sum of the first, third, and fifth terms of a geometric sequence is 63. The difference of the fourth and second terms is 9 times the ratio. Write down the first five terms of the sequence.

6. Let $x > 0$, $y > 0$, and $x + y = 2$. Prove that
$$\left(2 + \frac{1}{x}\right)\left(2 + \frac{1}{y}\right) \geq 9$$

7. The angle at vertex C of the acute triangle ABC is $45°$. Prove that the distance of the orthocenter of the triangle from the vertex C is equal to the side AB.

8. Two adjacent vertices of the parallelogram $ABCD$ are $A\,(8, 9)$ and $B\,(0, 3)$. A third vertex of the parallelogram is on the line $x + 2y = 6$. The area of the parallelogram $ABCD$ is 80 square units. Find the coordinates of the vertices C and D.

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