# Economics Entrance Exam - 1988

translated from: KoMaL, October 1988, pages 301-302
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These problems were given at the written mathematics entrance examination of the universities and colleges of economics in Hungary in 1988. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

*For all applicants*

1. Solve the given equation over the real numbers.

$$\sqrt{1+x\sqrt{x^2-24}}=x-1$$

(9 points)

2. The sum of the first three terms of an arithmetic sequence is $105$. If we leave the first two numbers unchanged and add $180$ to the third number, then we get the first three terms of a geometric sequence. Find the first three terms of the arithmetic sequence. (11 points)

3. Which real numbers satisfy the given equation?

$$\log\left(4^{x-2}+9\right)-\log\left(2^{x-2}+1\right)+\log 2-1=0$$

(12 points)

4. One angle of a triangle is equal to the sum of the other two angles. The radius of the circle circumscribed around the triangle is $6$ units, and the area of the square drawn on the shortest side is equal to the area of the triangle. Find the area of the triangle. (12 points)

5. In triangle $ABC$, $AB=BC$. Let $F$ be the midpoint of $BC$, and let $T$ be the foot of the altitude to the side $BC$. Find the coordinates of the vertices $B$ and $C$, if $A\,(-2,1)$, $T\,(16,1)$, and the second coordinate of $F$ is $13$. (14 points)

*Recommended for students from academic high schools*

6. Let $x$, $y$, $z$ be real numbers such that $xyz=1$ and $1+z+zx\neq 0$. Prove that the expression

$$\frac{1}{1+x+xy}+\frac{1}{1+y+yz}+\frac{1}{1+z+zx}$$

is defined, and its value does not depend on the choice of $x$, $y$, $z$. (12 points)

7. The tangents of the angles of a triangle are in the ratio $1:2:3$. What is the ratio of the sides of the triangle? (15 points)

8. Let $P$ be an arbitrary point in the plane of the quadrilateral $ABCD$. The reflection of $P$ through $A$ is $Q$, and the reflection of $P$ through $B$ is $R$. Prove that the quadrilateral $ABCD$ is a parallelogram if and only if the reflection of $Q$ through $D$ is the same point as the reflection of $R$ through $C$. (15 points)

*Recommended for students from vocational secondary schools*

6. In rectangle $ABCD$, $AB=2a$ and $BC=a$. Circles are drawn on the sides $AB$ and $AD$ as diameters; their intersection point inside the rectangle is $M$. Show that $M$ is on the diagonal $BD$. How long are the segments $MA$, $MB$, and $MD$? (13 points)

7. Solve the given equation over the real numbers.

$$x^2 + 2x \sin xy + 1 = 0$$

(14 points)

8. Prove that if the sum of the real numbers $a$ and $b$ is 1, then

$$a^4 + b^4 \geq \frac{1}{8}$$

(15 points)


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