Written Final and Entrance Exam - 1988

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These problems were given at the joint written final (érettségi) and entrance examination in mathematics in Hungary in 1988. Problems 1–5 were the same for all students; for problems 6–8, there was one version (A) recommended for students from academic high schools and another one (B) for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all students

1. Solve the given system of equations over the real numbers.

$$\frac{4}{x}+\frac{5}{y}=23, \qquad \frac{5}{x}-\frac{4}{y}=-2$$

(9 points)

2. Solve the given equation over the real numbers.

$$\left(\frac{3}{4}\right)^{\log x}+\left(\frac{4}{3}\right)^{\log x}=\frac{25}{12}$$

(11 points)

3. A rhombus with a 30° angle is drawn around a circle of area t. Express the area of the rhombus in terms of t. (12 points)

4. For which real numbers x are the following expressions defined?

a) $\sqrt{3-\sqrt{x-2}}$ b) $\sqrt{x-\sqrt{x-2}}$ c) $\sqrt{1-\tan^2 x}$

(13 points)

5. In triangle ABC, the angle at vertex A is twice the angle at vertex B, twice the side BC is equal to three times the side AC, and the area of the triangle is 27 times the sine of the angle at vertex C. Find the angles and the sides of the triangle. (13 points)

A. Recommended for students from academic high schools

6. Find the equation of the circle whose center is on the x-axis, and which is tangent from the inside to the circle k_1: $x^2+y^2+4x+6y-87=0$, and tangent from the outside to the circle k_2: $x^2+y^2-20x-12y+111=0$. (13 points)

7. The base of a pyramid is a square, and its lateral faces are congruent isosceles triangles. The height of the pyramid makes a 30° angle with the lateral faces. A cube is placed inside the pyramid so that four vertices of the cube are on the base of the pyramid, and the other four vertices are on the lateral faces, on the altitudes of the lateral faces drawn from the apex of the pyramid. Find the ratio of the volumes of the pyramid and the cube. (14 points)

8. Let p and q be positive integers with $q \geq p$. Prove that

$$\frac{q+\frac{1}{2}}{p-\frac{1}{2}} > \frac{1}{p}+\frac{1}{p+1}+\frac{1}{p+2}+\cdots+\frac{1}{q}$$

(15 points)

B. Recommended for students from vocational secondary schools

6. From the points P_1 and P_2 of the parabola $x - 2 = (y-1)^2$, the segment with endpoints $A\,(2,1)$ and $B\,(6,1)$ is seen at a right angle. Find the area of the quadrilateral AP_1BP_2. (13 points)

7. A right circular cylinder has height 2 and radius 7. A square is inscribed in the cylinder so that its plane intersects the axis of the cylinder, and its vertices are on the circles of the base and of the top. Find the side of the square and the angle between the axis of the cylinder and the plane of the square. (14 points)

8. Solve the given inequality over the real numbers.

$$2\sqrt{|1-2x|} \geq 1 + x$$

(15 points)

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