

Problem Set II - 1988 (January 1988)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following over the real numbers.

$$\text{a) } \frac{4x}{|x-1|} = 2x \qquad \text{b) } \frac{x+2}{2x-2} > -3$$

2. The sides of a triangle form a geometric sequence with ratio $\frac{4}{3}$. Find the largest angle of the triangle.

3. Find the equation of the circle that passes through the centers of the following three circles.

$$x^2 + y^2 = 4; \quad 3x^2 - 18x + 3y^2 + 12y = 36; \quad x^2 + y^2 + 10x + 4y + 12 = 0$$

4. For which real numbers x and y are both of the following equations true?

$$\sqrt{5^{x-y}} = \left(\frac{1}{5}\right)^{x-2y}; \quad \log_4(x+y) + \log_4(x-y) = 2$$

5. The base edge of a right pyramid with a square base is 4 units, and its height is 8 units. We place a hemisphere into the pyramid so that the circle bounding the hemisphere is in the base of the pyramid, and the hemisphere touches the lateral faces of the pyramid. Find the ratio of the volumes of the pyramid and the hemisphere.

Option A

- 6/A. Consider the parabola $y = 4 - x^2$, and all the rectangles that have one side on the x -axis and the two opposite vertices on the parabola. Find the sides of the rectangle with the largest perimeter.

- 7/A. We draw the median AP and the interior angle bisector AQ of triangle ABC , where P and Q are on the side BC . The circle through A, P, Q intersects the side AB at E and the side AC at F . Prove that $BE = CF$.

- 8/A. Solve the given equation over the real numbers.

$$\cot \frac{2\pi x}{x^2 + x + 1} = \frac{-\sqrt{3}}{3}$$

Option B

- 6/B. For what value of p is the sum of the squares of the roots of the equation $x^2 + (3p-2)x - 7p - 1 = 0$ the smallest?

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- 7/B. A circle with center C is intersected by a parabola whose vertex is the center of the circle and whose focus F is on the circle. Let A and B be the intersection points of the parabola and the circle, and let E be the intersection point of the line AB and the axis of symmetry of the parabola. Show that the segment DE is the geometric mean of the segment FE and the diameter, where D is the point of the circle opposite to F .
- 8/B. For which positive integers N is $N + 1987$ a divisor of $N^2 + 1987$?