Technical Universities Entrance Exam - 1988

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These problems were given at the written mathematics entrance examination of the technical universities and colleges in Hungary in 1988. Problems 1–5 were the same for all applicants; for problems 6–8, there was one version recommended for students from academic high schools and another one for students from vocational secondary schools. The points for each problem are shown in parentheses.

For all applicants

1. Find the exact values of the given expressions without using approximate values.

$$A = 10^{1-\log 2} + 3^{\log_9 36}, \qquad B = 125^{-\frac{1}{3}} \cdot \sqrt[3]{1000} \cdot \log_{\sin\frac{\pi}{6}} \sqrt{2}, \qquad C = \sqrt{\log\tan 36^\circ + \log\tan 54^\circ}$$

(9 points)

2. Let AB be a diameter of a circle of radius 1, and let C be the point on the tangent line to the circle at A for which the segment AC has length $2\sqrt{3}$. Find the area of the common part of the triangle ABC and the disk. (11 points)

3. We build a large cube out of small cubes with unit edges. If we build the largest possible cube from the small cubes we have, then 107 small cubes are left over. If we want one more small cube along each edge, then we would need 62 more small cubes. How many small cubes do we have? (12 points)

4. For which pairs of real numbers is the following system of equations true?

$$\begin{aligned} y - x &= 44 \\ \sqrt{\frac{6x}{x+y}} + \sqrt{\frac{x+y}{6x}} &= \frac{5}{2} \end{aligned}$$

(13 points)

5. The line $x + y = 30$ intersects the x-axis at A and the y-axis at B, and let O be the origin. Find the equation of the line e that is parallel to the line $x - 3y = 6$, intersects the segment OB at C and the segment AB at D, and for which the area of the quadrilateral $OADC$ is 234 units. (13 points)

Recommended for students from academic high schools

6. For given real numbers p and q, find the first term and the difference of every arithmetic sequence in which the sum of the first four terms is p, and the quotient of the fourth term and the first term is q. (13 points)

7. Solve the given equation, if p is a prime number and n is a positive integer.

$$1 + p + p^2 + \cdots + p^n = 2801$$

(14 points)

8. Consider four vertices of a unit cube such that no two of them are on the same edge of the cube. The tetrahedron determined by these four points is cut by planes parallel to two opposite edges of the tetrahedron. Considering all such planes, what are the largest possible perimeter and the largest possible area of the cross-section? (15 points)

Recommended for students from vocational secondary schools

6. The difference of the volumes of two right circular cylinders is 48π, and the difference of their lateral surface areas is 24π. The radius of the cylinder with the larger volume is 1 more than the radius of the other one, and its height is $\frac{4}{3}$ times the height of the other one. Find the heights and the radii of the cylinders. (13 points)

7. Solve the given equation over the pairs of real numbers.

$$2\cos^2\frac{x^2+3y}{6}=3^x+3^{-x}$$

(14 points)

8. In a geometric sequence of positive numbers, the sum of the first n terms is S, and the sum of the reciprocals of the first n terms is R. Express the product of the first n terms in terms of S, R, and n. (15 points)

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