

Problem Set I - 1989

author: Imre Rábai

translated from: KoMaL, September 1989, pages 253-254

Pdf file: KoMaL, September 1989, pages 253-254

In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given equation.

$$(x - 2)^2 = \sqrt{4 - 4x + x^2}$$

2. The diagonals of an isosceles trapezoid are perpendicular to each other, and they divide each other into two parts whose ratio is 7 : 17. Find the area of the trapezoid, if its perimeter is $100\sqrt{2}$ units.
3. Solve and discuss the following system of equations, where a is a real parameter.

$$\begin{aligned} ax + 3y &= 12 \\ 12x + ay &= 24 \end{aligned}$$

4. In the isosceles triangle ABC , $AB = AC = 4$ units. The area of the triangle is 4 square units. Find the angles of the triangle and the length of the side BC .
5. Find the value of the parameter a so that one root of the equation

$$x^2 - ax + 2a - 4 = 0$$

is three times the other root.

6. One side of a trapezoid is 8 units long, and the sum of the side parallel to it and the height is 12 units. How should we choose the height of the trapezoid so that its area is as large as possible? What is the largest area?
7. Two congruent circles that are tangent to each other are placed between two parallel lines that are 8 units apart. (The circles are also tangent to the lines.) The distance between the lines through the centers of the two circles perpendicular to the given parallel lines is 4 units. Find the radius of the circles.
8. The orthocenter of the acute triangle ABC is M , and the point of the circumscribed circle opposite to A is A' . Prove that the quadrilateral $BMCA'$ is a parallelogram.