

Problem Set II - 1989(November 1989)

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the given system of equations.

$$\begin{aligned}x^2 + 3xy &= -2 \\ 2xy - y^2 &= -5\end{aligned}$$

2. One root of the equation $x^2 + px + q = 0$ is 3, and the other root is equal to twice the discriminant of the equation. Find p and q .

3. Solve the given equation.

$$\log(4 - x) + 2 \log 3 = \log 108 - \frac{1}{2} \log x^2$$

4. The radius of the circle circumscribed around triangle ABC is 5 units, the side AB is 8 units long, and the ratio of the other two sides is 2 : 5. Find the other two sides of the triangle.

5. Find the equation of the circle that is tangent to the x -axis at the origin and is also tangent to the line $y = x + 2$.

6. Where does the function

$$f(x) = \sin^2 2x + 2 \cos^2 x - \frac{5}{4}$$

take its largest and its smallest values on the interval $[0, \pi]$? What are the largest and smallest values?

7. Depending on the value of the real number p , how many solutions does the equation

$$\sqrt{2 |\log_2 x| - (\log_2 x)^2} = p$$

have?

8. The sides a , b , and c of a triangle satisfy the following relation.

$$\frac{1}{a+b} + \frac{1}{b-c} = \frac{3}{a+b-c}$$

Find the angle opposite the side b .