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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The midline of a tangential trapezoid (a trapezoid with an inscribed circle) divides it into two parts whose areas are in the ratio 5 : 11. Find the lengths of the parallel sides of the trapezoid, if the legs are 3 and 5 units long.
2. Solve the following equations over the real numbers.
 - a) $2 \log_3 (x - 2) + 2 \log_3 (x - 4) = 0$
 - b) $2 \log_3 (x - 2) + \log_3 (x - 4)^2 = 0$
 - c) $\log_3 (x - 2)^2 + \log_3 (x - 4)^2 = 0$
3. Find the equation of the circle whose tangent line at its point $A(3, 2)$ is $x + y = 5$, and whose center is 1 unit away from the origin.
4. Find the value of m if the sum of the squares of the real roots of the equation

$$x^2 + (m - 2)x + m + 5 = 0$$

is 10.

5. The product of the third and fifth terms of a geometric sequence is 9, and the sum of the second and sixth terms is $-\frac{51}{4}$. Find the first term and the ratio of the sequence.
6. In trapezoid $ABCD$, one of the parallel sides is $BC = 4$ units, the legs are $AB = 2\sqrt{3}$ and $DC = \sqrt{10}$ units, and the angle BAD is 60° . Find the area of the trapezoid.
7. The areas of the base and the top of a frustum of a pyramid are t and T . We cut the frustum with the plane that is halfway between the base and the top (parallel to them). Express the area of the cross-section in terms of t and T .
8. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}\cos y &= \sqrt{2} \cos x \\ \sin y &= \sqrt{\frac{2}{3}} \sin x\end{aligned}$$