

author: Imre Rábai

translated from: KoMaL, October 1990, page 295

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Find the exact values of the following expressions.

$$\text{a) } \frac{\log_3 90}{\log_{30} 3} - \frac{\log_3 270}{\log 3} \qquad \text{b) } \frac{\left(1 - \tan^2 \frac{\pi}{8}\right) \left(2 \sin^2 \frac{5\pi}{12} - 1\right)}{2 \tan \frac{\pi}{8}}$$

2. The product of the second and sixth terms of a geometric sequence is 576, and the difference of the fifth and third terms is 36. Write down the first six terms of the sequence.
3. Find the equation of the line through the point $P(2, 2)$ that is tangent to the circle $x^2 + y^2 = 2x$.
4. The area of a triangle is 40 square units, and two of its medians are 5 and 12 units long. How long is the third median?
5. The sides of a triangle are a, b, c , and the opposite angles are α, β, γ . Find the exact value of $\sin \beta$ if $a + c = 2b$ and $\gamma = \alpha + 90^\circ$.
6. The segment AA' is a diameter of the circle circumscribed around triangle ABC . Reflect A' through the midpoint A_1 of the side BC . Prove that this reflection is the orthocenter of the triangle.
7. Solve the given equation over the real numbers.

$$2^{2x+1} = 3 \cdot 2^x + \sqrt{4 - 2^{x+3} + 2^{2(x+1)}}$$

8. For what values of the real parameter m does the equation

$$\frac{2m \cdot \cos 2x - 4(m-1) \cdot \cos x + 3m - 2}{1 - 2 \cdot \cos x} = 0$$

have a real solution?