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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Which one is greater?

a) $\sqrt{4 - 2\sqrt{3}} - \sqrt{4 + 2\sqrt{3}}$ or $\frac{\tan \frac{2\pi}{3}}{\sin\left(-\frac{5\pi}{3}\right)}$

b) $2 \log_2 5$ or $3 \left(1 + \log_{\frac{1}{8}} \frac{1}{3}\right)$

2. Prove that if the angles α, β, γ of a triangle satisfy

$$\frac{\sin^2 \alpha}{\sin 2\alpha} = \frac{\sin^2 \beta}{\sin 2\beta} = \frac{\sin^2 \gamma}{\sin 2\gamma}$$

then the triangle is equilateral.

3. Solve the following equations over the real numbers.

a) $\frac{x-3}{\sqrt{x+1}-2} = \sqrt{x+1} + 2$

b) $\log(4-x^2) - \log(2+x) = \log(2-x)$

c) $\frac{\sin \alpha - \frac{1}{2} \sin(\pi + 2\alpha)}{1 + \cos \alpha} = \sin \alpha$

4. Solve the system of equations

$$\begin{aligned} x - y &= a + 1 \\ 2xy + 2y &= -a^2 - 4 \end{aligned}$$

where a is a real parameter.

5. The perimeter of a right triangle is 15 units, and the radius of its inscribed circle is 1 unit. Find the sides of the triangle.
6. In rectangle $ABCD$, $3 \cdot AB = 2 \cdot BC$. Find the coordinates of the points C and D , if $A(-2, -2)$ and $B(4, 2)$.
7. In the arithmetic sequence (a_n) , $a_3 = 3a_2$ and $s_n = 40a_3$, where s_n is the sum of the first n terms. Find the n th term of the sequence.

8. Solve the following inequalities over the real numbers.

a) $2^{\sqrt{x+2}} < 3^{\sqrt{5} \cdot \log_3 2}$

b) $\log_{1/2} (x^2 - 3x - 2) < -1$

c) $\sin(\pi - x) \cdot \sin\left(\frac{3\pi}{2} + x\right) > \frac{\sqrt{3}}{4}$