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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following system of equations.

$$\begin{aligned}\sin x &= 2 \sin y \\ x + y &= \frac{2\pi}{3}\end{aligned}$$

2. Solve the following inequalities.

a) $\sqrt{x^2 - 5x + 4} > -1$

b) $\log_{\frac{1}{2}}(4^x - 5 \cdot 2^x + 8) < -2$

3. In triangle ABC , $AB = \sqrt{3}$, $AC = \sqrt{2}$ units, and $\angle BAC = 75^\circ$. On the arc BC of the circumscribed circle that does not contain A , mark the point D so that $\angle BAD = 30^\circ$. Find the length of the segment AD .
4. The difference of an arithmetic sequence is $\frac{1}{2}$. The sum of the first n terms is 38, and the sum of the first $n + 4$ terms is 69. Find the value of n and the first term of the sequence.
5. The volume of the sphere inscribed in a right circular cone is one third of the volume of the cone. What fraction of the surface area of the cone is the surface area of the sphere?
6. A parabola with axis parallel to the y -axis passes through the point $A(4, -7)$ and is tangent to the line $y = 1$. A normal vector of the tangent line to the parabola at A is $\mathbf{n}(8, 1)$. Find the equation of the parabola.
7. Solve the equation

$$\sqrt{x^2 + 3x + 2 + \frac{p^2}{x^2}} = x + 1 - \frac{p}{x}$$

where p is a real parameter.

8. The perimeter of triangle ABC is $2s = 24$ units. We draw the tangent lines to its inscribed circle that are parallel to the sides. Of the segments of these tangent lines inside the triangle, choose the longest. For which triangles ABC is this tangent segment as long as possible?