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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. In the regular hexagon $ABCDEF$, let G be the midpoint of the side BC , and H the midpoint of the diagonal DF . Prove that the triangle AGH is equilateral. What is the ratio of the areas of the triangle and the hexagon?
2. For the four-digit number $\overline{abba}$, we know that $27(\overline{ba} - \overline{ab}) + 47 = \overline{abba}$. What is this four-digit number?
3. For which x is the expression $\sqrt{\log(x^2 + 2x - 15)}$ defined?
4. A sequence of points is given in the coordinate system: $A_1(1, 1)$, $A_2(0, 2)$, $A_3(2, 2)$, $A_4(4, 0)$, and we also know that if $A_n(a, b)$, then $A_{n+4}(4a, 4b)$. How long is the broken line $A_1A_2 \dots A_{101}$?
5. The volume and the surface area of a regular square prism are equal (as numbers), and every edge has integer length. Find all such prisms.
6. We add two angles of a quadrilateral (measured in radians), add the other two angles, and multiply the two sums. Show that the product is at most π^2 .
7. In triangle ABC , T is the foot of the altitude from C . CAQ and CBP are right triangles with the right angles at A and B , such that $AQ = BT$ and $BP = AT$. Prove that $CP = CQ$.
8. Find the values of the parameter p for which the following equation has no real root.

$$\sin^4 x + \cos^4 x = p$$