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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Find the exact value of the following expression.

$$\frac{(\log_9 27 - \log_4 2) \left(\sqrt{27} - 3\sqrt{8} - \frac{1}{3}\sqrt{243} \right)}{\sin \frac{7\pi}{4} - \cos \frac{7\pi}{4}}$$

2. The area of a right trapezoid circumscribed around a circle of radius $r = 2$ units is 18 square units. Find the lengths of the sides of the trapezoid.

3. Solve the equation

$$10 \cos^2 x - 16 \sin x = \cos 2x + 15$$

over the real numbers.

4. Find the equation of the circle that passes through the points $A(1, 2)$ and $B(4, 5)$, and whose center is on the line $2x - y = 6$.

5. Solve the equation

$$\sqrt{2 + \log_x 4} = -\frac{\sqrt{12}}{\log_2 x}$$

over the real numbers.

6. The tangents of the angles of a triangle are in the ratio $2 : 3 : 4$. What is the ratio of the sides of the triangle?
7. On the side AB of the square $ABCD$ as hypotenuse, we draw right triangles outward. Prove that the bisectors of their right angles all go through one point and bisect the area of the square.
8. Find all values of the real parameter p for which the inequality

$$4x^2 + 4y^2 + pxy - x - y + \frac{1}{16} \geq 0$$

holds for every pair of real numbers (x, y) with $x^2 = y^2$.