

author: Imre Rábai

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The medians to the legs of a right triangle are 3 and 4 units long. Find the length of the median to the hypotenuse.
2. A circle through the vertices A and C of triangle ABC intersects the side AB at D and the side BC at E , so that

$$CE = 1, \quad CD = BE = 4, \quad \text{and} \quad AD = 4BD$$

Find the area of the triangle.

3. Solve the following inequality over the real numbers.

$$\log_{x+1} (2x^2 - 3x + 1) \leq 2$$

4. Solve the following equation over the real numbers.

$$\sqrt{16 \cos^4 \frac{x}{2} - 7 \cos^2 \frac{x}{2} + \sin^2 \frac{x}{2}} = 2 \cos x + 1$$

5. The base of the pyramid $ABCDE$ is the rectangle $ABCD$, with $AB = 5$, $BC = 2$, $EA = 3$, $EB = 4$ units. Find the lengths of the edges EC and ED of the pyramid if the volume of the pyramid is as large as possible. What is the volume of the pyramid?
6. The line $x + y = 20$ intersects the x -axis at A and the y -axis at B , and let O be the origin. Find the equation of the line e that is parallel to the line $x = 3y$, intersects the segment OB at C and the segment AB at D , and for which the area of the quadrilateral $OADC$ is 104 square units.

7. Solve the equation

$$(2x)^2 - 4x\sqrt{m \cdot 2^m} + 2^{m+1} + m - 2 = 0$$

over the real numbers, where m is a real parameter.

8. Solve the following system of equations over the triples of real numbers.

$$\begin{aligned} x^2 &= y + z + 2 \\ y^2 &= z + x + 2 \\ z^2 &= x + y + 2 \end{aligned}$$