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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations.

a) $\cos |x| = |\cos x|$ b) $\log |x| = |\log x|$ c) $\frac{\sin(-x)}{\sin x} = -1$

2. Explain why the following is true.

$$\sin 16^\circ + \sin 28^\circ + \sin 62^\circ + \sin 74^\circ = \sqrt{2} \sin 61^\circ + \sqrt{2} \sin 73^\circ$$

3. The equation of the midline of triangle ABC parallel to the side AB is $k: x - 2y + 6 = 0$, its centroid is $S\left(\frac{5}{3}, \frac{4}{3}\right)$, the vertex C is $(-1, 10)$, and another vertex is on the x -axis. Find the coordinates of the missing vertices.

4. Prove that

$$\frac{1}{p \cdot q} = \frac{1}{a^2} + \frac{1}{b^2}$$

where a and b are the legs of a right triangle, and p and q are the projections of the two legs onto the hypotenuse.

5. On the sides of triangle ABC , we draw similar plane figures, whose areas are t_a , t_b , and t_c . Prove that $t_b + t_c - t_a = 2 \cdot \sqrt{t_b \cdot t_c} \cdot \cos \alpha$, where α is the interior angle of the triangle opposite the side of t_a .

6. Solve the following system of equations over the real numbers.

$$\begin{aligned} x + y &= 1 \\ x^5 + y^5 &= 2 \end{aligned}$$

7. A cone standing on its apex (with base radius r and height m) is filled with water up to a certain height, so that when the cone is turned onto its base, the water is half as high as the cone. How high was the water?
8. For a non-constant arithmetic sequence, dividing the sum of the first n terms by the sum of the next n terms gives the same quotient for every n . What is this quotient?