

author: Imre Rábai

translated from: KoMaL, April 1992, page 149

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. How many sides does the regular polygon have in which the measure of an exterior angle in degrees is 18 less than the number of sides?
2. For which real numbers x are the following expressions negative?

a) $\frac{x^4 - 6x^2 + 8}{4 - x^2}$

b) $\log_{\frac{1}{2}} \cos x - 1$

c) $|x - 2| - |x + 2| + x$

3. The fifth term of an arithmetic sequence is 35, and the fiftieth term is 260. Leave out every term of the sequence whose last digit is 5. Express, in terms of n , the quotient of the sums of the first n terms of the left out sequence and of the remaining sequence (n is a positive integer). For which values of n is this quotient greater than $1 - \frac{1}{1003}$?
4. Three vertices of a trapezoid $ABCD$ are $A(2, -4)$, $B(-4, 0)$, $C(-1, 1)$. AB is one of the parallel sides of the trapezoid, and $BC = AD$. Find the coordinates of the fourth vertex of the trapezoid.
5. Solve the following system of equations over the real numbers.

$$\begin{aligned} x + 2y &= 1 \\ x^3y + 4xy^3 &= -410 \end{aligned}$$

6. Find the ratio of the surface areas of a right circular cone and the sphere inscribed in it, if the ratio of their volumes is 3 : 1.
7. Find the sides of a right triangle if they are natural numbers, and the perimeter and the area of the triangle are equal (as numbers).
8. Consider the function $x \mapsto (1 - p)x^2 - 2(2p + 1)x - 4p - 1$, where x is any real number and p is a real parameter. Find the value of p for which the largest value of the function is 8.