

author: Imre Rábai

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following system of equations over the pairs of real numbers.

$$\begin{aligned}x^2y + xy^2 &= -12 \\ \frac{1}{x} + \frac{1}{y} &= -\frac{4}{3}\end{aligned}$$

2. The parallel sides of an isosceles trapezoid are 8 and 48 units long. Its leg is equal to the sum of the shorter parallel side and the height. Find the area of the trapezoid and the radius of its circumscribed circle.
3. Solve the following inequality over the real numbers.

$$3 \cdot 4^{x+1} - 35 \cdot 6^x + 2 \cdot 9^{x+1} \geq 0$$

4. Find the equation of the circle through the point $P(9, 5)$, if it is tangent to the y -axis at a point Q for which $PQ = 3\sqrt{10}$.
5. The sides of a triangle are 4, 6, and 8 units long. Find the radius of the circle whose center is on the longest side and which is tangent to the other two sides. Find the length of the angle bisector to the longest side.
6. What are the largest and smallest values of the function $x \mapsto |x^2 - x| + |x^2 + 3x + 2|$ on the closed interval $\left[-\frac{3}{2}, \frac{1}{2}\right]$? Where does it take these values?
7. Twice one root of the equation $x^2 + px - q = 0$ is a root of the equation $x^2 - 3px + q = 0$. Find p and q if $p^2 + 4q = 36$.
8. Consider the equation $p \cdot \tan x = 1 - \cos 2x$, where p is a real parameter.
 - a) Solve the equation if $|p| = \frac{1}{2}$.
 - b) Depending on the value of p , how many solutions of the equation are in the closed interval $[0, \pi]$?