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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Solve the following equations over the real numbers.

a) $(2x^2 - x - 1)(x^2 - 1) = 0$ b) $(2x^2 - x - 1)^2 + (x^2 - 1)^2 = 0$

2. Prove the identity

$$\frac{1 + \tan \alpha}{1 - \tan \alpha} = \frac{1 + \sin 2\alpha}{\cos 2\alpha}$$

For which real numbers α is the identity true?

3. Solve the equation $x^{\log_3 x} = 9x$ over the real numbers.
4. Two sides of a triangle are 3 and 4 units long, and its area is $3\sqrt{2}$ square units. Find the third side of the triangle.
5. The orthocenter of triangle ABC is the origin, and $C(0, -7)$. Find the coordinates of the missing vertices and the area of the triangle, if the side AB goes through the point $P(0, 9)$ and its length is as short as possible.
6. In rectangle $ABCD$ the side AB is twice the side AD . The distances of a point P of the plane from the vertices A, B , and D are $PA = 2, PB = 4, PD = 3$ units. Find the area of the rectangle. Where is the point P ?
7. Find the value of the parameter p for which the largest value of the polynomial

$$f(x) = -x^2 + 4px - 4p^2 + 2p + 2$$

on the interval $[0, 3]$ is -2 .

8. The difference of two positive integers is k . If we add 3 times the larger number to 4 times the smaller number, we get 84. For which values of k does the problem have a solution, and what are the solutions?