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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets usually consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The roots of the equation $x^2 - 16x + 57 = 0$ are a and b . Without computing a and b , find the sum of the volumes of the cone with height a and base radius b and the cone with height b and base radius a .
2. The radius of the inscribed circle of a triangle is 4 units, and the radii of the excircles are 14, 12, and 10.5 units. Find the sides and the area of the triangle.
3. Find the value of the parameter p for which the following equation has a real root.

$$\sqrt{x^2 - 8x - 65} + \sqrt{x^2 - 169} + \sqrt{x^2 + (p + 14)x + 5p + 9} = 0$$

4. The second coordinates of all three vertices of the acute triangle ABC are positive, and one side is parallel to the x -axis. The equation of the circumscribed circle is $x^2 - 8x + y^2 - 8y - 8 = 0$, and the orthocenter is $(2, 6)$. Find the radius of the inscribed circle.
5. Solve the following equation over the real numbers.

$$\log_3^2(x^2 - 8x + 15) - [\log_3(x^2 - 8x + 15) - 1] \cdot \log_3 5 = \log_3(x^2 - 8x + 15) + [\log_3(x^2 - 8x + 15) - 1] \cdot \log_3 7$$

(Here $\log_3^2 u$ means $(\log_3 u)^2$.)

6. Find the pairs of real numbers for which

$$\sin^8 x + \sin^6 x + \sin^4 x + \sin^2 x + 1 = \sin^4 x (2^{1+\cos y} + 1)$$

7. On the circle with center at the origin and radius 1, find the points whose coordinates give the largest value of the expression $x^2 + xy + y^2$.