

author: Róbert Gyimesi

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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. For which real numbers x is it true that

a) $(x + 1) \sqrt{4 - x^2} \leq 0$

b) $\cos x \geq 2 \cos \frac{x}{2} + \cot^2 \frac{\pi}{6}$

c) $(3^{x-1})^x \leq \sqrt{5}^{\sin(1993\pi)} - \log_{1/\sqrt{2}} 16$

2. From a point outside a circle of radius r , we draw two secants to the circle: one through the center of the circle, and the other one at distance $\frac{r}{2}$ from the center. Find the perimeter and the area of the part of the disk bounded by the chords of the two secants and the arcs between them.

3. For a non-constant arithmetic sequence, the product of the first and second terms, the product of the second and third terms, and the product of the third and first terms are, in this order, three consecutive terms of a geometric sequence. What is the ratio of the geometric sequence?

4. About a point P in the plane of the equilateral triangle ABC we know that $PA = PB = 2\sqrt{13}$ and $PC = 2\sqrt{3}$. What is the area of the triangle?

5. Two sides of a symmetric tangential quadrilateral (a quadrilateral with an axis of symmetry and an inscribed circle) are on the lines $3x - 4y + 24 = 0$ and $y = \frac{3}{4}x - 4$, and two of its vertices are on the y -axis. What kind of quadrilateral is it, and what are the coordinates of its other vertices in the first quadrant?

6. For which values of the real parameter p are the following equations equivalent?

$$(1) \quad \sin 2x + p = \sin x + 2p \cos x$$

$$(2) \quad 2 \cos 2x + p^2 = 5p \cos x - 2$$

7. At most what fraction of the volume of a frustum of a right circular cone can be filled by the sphere inscribed in the frustum (the sphere that touches both bases and the lateral surface)?

8. For which pairs of real numbers (x, y) is it true that

$$5x^2 + 8xy + 4y^2 = 4 + 4x \quad \text{and} \quad \cos^2(\pi x) - \sin^2(\pi y) = 1?$$