Problem Set - 1993 (December 1993)

author: Imre Rábai
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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.
The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.
In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).
This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. The midline of the isosceles trapezoid $ABCD$ is 8 units long, and its height is 4 units.

   a) Find the area of the trapezoid.

   b) Find the perimeter of the trapezoid and the radius of its circumscribed circle, if the diagonal AC is perpendicular to the leg BC.

2. For what values of the parameter k does the quadratic equation

$$x^2 - 2(3k+2)x + k^2 + 11k - 102 = 0$$

   have two real roots with opposite signs?

3. Solve the following equation on the interval $0 \leq x \leq 2\pi$.

$$\sin 2x + |\sin x| + \sin\frac{11\pi}{2} = \cos 2x - 2\cos^2 x$$

4. For which real numbers is the following inequality true?

$$\log_{\frac{1}{2}}\sqrt{x+2} < 1 + \log_{\frac{1}{2}}\sqrt{9-x^2}$$

5. Find the values of the variables for which the following expressions take their largest values, and find these largest values.

   a) $\dfrac{16}{2^{x+2}+2^{-x}}$ b) $-x^4 + 8x^2 - 12 - (1 + x + \log_2 y)^2$

6. Prove that if the angles α, β, γ of a triangle satisfy $\dfrac{\sin\gamma}{\cos\beta} + \dfrac{\sin\beta}{\cos\gamma} = 2\sin\alpha$, then the triangle is a right triangle.

7. The circle k is tangent to the x-axis and to the line e with equation $3x - 4y + 45 = 0$; the second coordinate of the common point of k and e is 9. Find the equation of the circle.

8. Prove that if n is a positive integer, then

$$1\cdot 2\cdot 3 + 2\cdot 3\cdot 5 + \cdots + n(n+1)(2n+1) = \frac{1}{2}n(n+1)^2(n+2)$$

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