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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. Two vertices of an isosceles triangle are the points $(1, 6)$ and $(-5, 2)$. What relation holds between the coordinates of the points that can be the third vertex?
2. The third term of a geometric sequence of positive integers is 6 more than the first term. What is the sum of the first three terms of the sequence?
3. An isosceles triangle whose legs make a 120° angle with each other is circumscribed around a circle of radius r . Find the exact values of the perimeter and the area of the triangle.
4. Which pairs of real numbers (x, y) satisfy the following system of equations?

$$\begin{aligned}\log_x y + \log_y x &= \frac{5}{2} \\ 4\sqrt{x} - 3\sqrt{y} &= 1\end{aligned}$$

5. In the plane of the isosceles right triangle ABC (with the right angle at C), the point P is chosen so that $AP = BP = \sqrt{5}$ and $CP = 1$. What is the area of the triangle ABC ?
6. Find the largest subset of the real numbers on which the following expression is defined.

$$\frac{\sqrt{3 \cos x - \cos 2x - 2}}{\log(2 - x - x^2)}$$

7. We cut the cube $ABCD A' B' C' D'$ with the plane through the midpoint of the edge BB' and the vertices A and D' . What is the ratio of the volumes of the two parts of the cube?
8. The diagonals of the quadrilateral $ABCD$ inscribed in a circle of radius 1 intersect at M . What can we say about the area of the quadrilateral, if we know that the product of the areas of the triangles ABM and CDM is $\frac{1}{4}$?