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In 1983, a new system of entrance examination was adopted. In order to be accepted to a university, the 3rd and 4th years of high school grades in mathematics, Hungarian language and literature, history, foreign language, physics, (biology, chemistry, geography, another foreign language - students choose from these) are counted toward university entrance performance.

The 'brought' points (i.e. the points comprised of final grades in high school in the subjects listed above) add up to a total of 60 points. In addition, students are assessed by written and oral examinations for a total of 60 points. So, there is a total of 120 points possible.

In Mathematics, the same exam serves as the GED and the university entrance exam. These problem sets consist of 8 problems, presented in order of difficulty (from easiest to most difficult).

This problem set is similar to such an exam. We advise the reader to work through the problem set while measuring the time. There are 180 minutes to solve and present all problems.

1. One leg of a right triangle is 6.5 units long, and the altitude to the hypotenuse is 6 units. Find the other leg and the hypotenuse.
2. In triangle ABC , $AB = 1$ unit, $\angle BAC = 45^\circ$, and $\angle ABC = 120^\circ$. Find the exact lengths of the sides AC and BC .
3. Of five numbers, the first four are consecutive terms of an arithmetic sequence, and the last three are consecutive terms of a geometric sequence. Find these five numbers, if the sum of the first four is -36 , and the product of the second and third is 72.
4. The coordinates of the point P are $(4, 1)$, the equation of the line e is $x - y = -1$, and the equation of the line f is $x + 2y = 11$. Find the equation of the line through P whose segment between e and f is divided by P into thirds, with P being the point of division closer to f .
5. Solve the equation $\sqrt{4 \cdot \log_2 x - (\log_2 x)^2} = p$ over the real numbers, where p is a real parameter.
6. Solve the following inequality over the real numbers.

$$\frac{x^2}{(1 - \sqrt{1 - x})^2} < 9 - x$$

7. Prove that if α, β, γ are the angles of a triangle, then

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + 2 \cos \alpha \cos \beta \cos \gamma = 1$$

8. Find the integers k and n for which the roots of the equation

$$(2k - 1)nx^2 + (2k - 1)n(k - n - 4)x - 2(2k - 1)n(k - n - 2) - 1 = 0$$

are also integers.